

# Implementation of a Fully Coupled Reynolds-Structural Analysis in an Open Source Finite Element Framework

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## Overview

- Motivation and Background
- Goals
- Brief Overview of OOFEM
- General Approach and Some Details
- Some Results
- Future Work
- Summary/Observations



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## Motivation – Big Picture

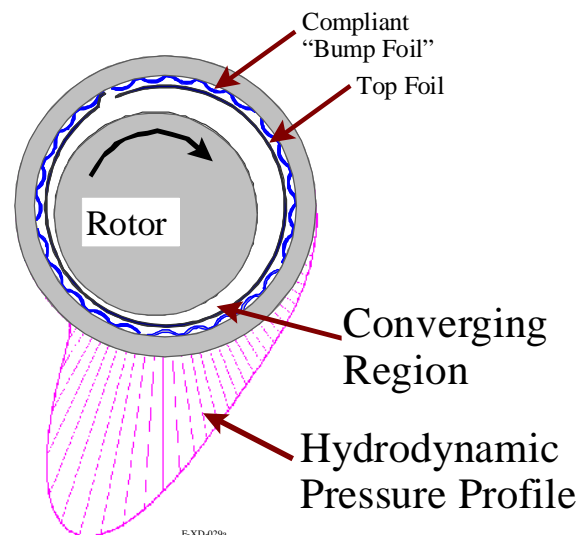
If Xdot wants to  
develop and sell  
foil bearings...

**WE NEED A CODE!**

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## Foil Bearings

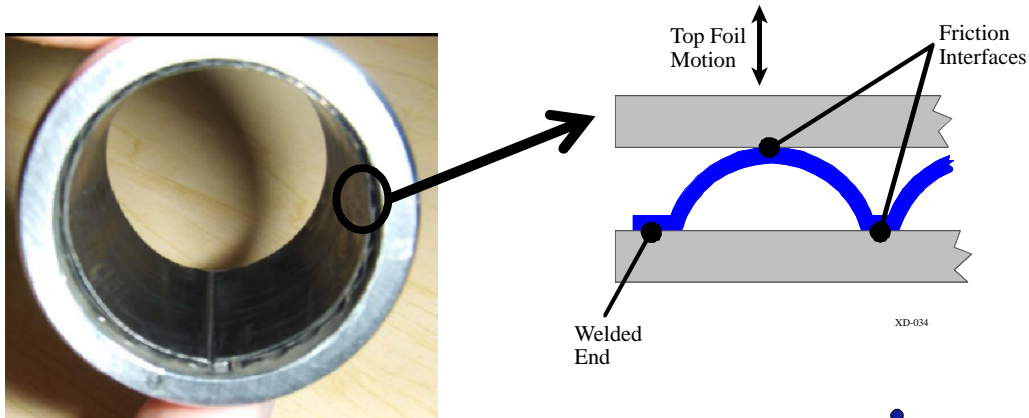
- Hydrodynamic bearings that combines a compliant operating surface with a support structure that provides stiffness and damping (like a squeeze film damper/bearing in a way...)



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## Compliant Structure

- One common design is “bump foil”
- Friction is significant source of damping
- Foil deflection can be much larger than film thickness
- Physics couple air film to structural deflections



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## Foil Bearings

- Work well for (very) high shaft speeds (50k~120k RPM)
- Capable of operating at extreme temperatures (no oil)
  - 1200 F
  - Cryogenic
- Generally much better performance than rigid pad air bearings
- Typical applications
  - Smaller (< 200 kW) turbomachinery
  - Air cycle machines for aircraft environmental control,
  - Cryogenic Expanders
  - Microturbines
  - High speed, direct drive, oil-free compressors

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# Analysis Code Goals

- Need to model both fluid and compliant structural components
  - Compressible Reynolds equation is a reasonable approach for fluid portion for air bearings
  - Compliant structural component can be modeled with shell and or beam elements
- Code that is reasonably efficient and converges for a wide range of cases
  - Design tool, not a research tool

# Goals

- Need to predict stiffness and damping coefficients
  - Will be frequency dependent since air is compressible
  - Need to include effect of both fluid and structure
- Analysis that is modular and extensible
  - Radial and Thrust
  - Various spring support structure configurations
  - Ultimately also want a thermal model (energy equation)
    - Viscosity changes with temperature less of an issue for air than for oil
    - Thermal management is a key for successful foil bearing operation

## Some Possible Approaches

- Develop and write a completely new code
  - Gives total control
  - Quite a lot of work
- Use commercial multiphysics like Comsol
  - Mature, solid analysis with lots of options
  - Quite pricy
  - Not clear that it would be easy to get stiffness and damping
  - Run-times will probably be long
- Use an existing open-source finite element framework
  - Can leverage previous work by others to speed development (SBIR Funding about 6 months)
  - Framework would probably will have structural analysis already
  - Seemed very interesting

## OOFEM (Object Oriented Finite Element Method)

- Considered 11 different open source frameworks
- Selected OOFEM
- Initially developed by a Czechoslovakian civil engineering professor
- Fairly mature and stable code (under development since 1997)
  - Modular, extensible
  - Variety of analysis implemented as examples
  - Active user/developer forum
- Written in C++
- Had a basic shell element already implemented

# OOFEM

- Strong structural analysis capability
  - Linear, nonlinear analysis
  - Wide range of elements and material models
- Module(s) for Heat Transfer analysis
- Module(s) for Incompressible Flow analysis
- “Bookkeeping” aspects are fairly problem independent
- Variety of numerical solvers
- Support for
  - Parallel solution
  - Adaptive solutions
  - eXtended Finite Element implementation

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# OOFEM

- Appeared suitable for coupled analysis implementation
  - Implementing mixed fluid-structural elements/computational domains appeared feasible
- Some significant drawbacks/issues
  - Pretty complex code
  - Clear that there would be a steep learning curve
  - Implementing a coupled analysis was going to require some significant development work, not clear it would be possible
  - Developed under Linux, I'm more comfortable in a Windows based compiler/environment
- Decided that potential benefits outweighed the downsides

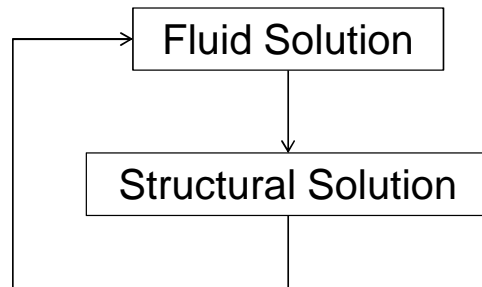
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# Coupled Reynolds-Structural Analysis

## ■ Sequential-Iterative

- Solve each problem separately, iterate back and forth
- Can even do with two completely separate codes and some “glue”
- Convergence can be difficult to achieve
- Not clear how you would get stiffness and damping coefficients, since the physics are so tightly coupled



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# Coupled Reynolds-Structural Analysis

## ■ Fully Coupled (including shaft location)

- Include all coupled variables in the nonlinear solution at once
- Potentially more efficient, tradeoff of larger matrix inversion versus maybe fewer iterations
- Obtain stiffness and damping from a perturbation solution
- “Easy” extension to misalignment and moment coefficients
- “Easy” extension to pad degrees of freedom, etc. for conventional bearings
- Extension to include thermal effects (energy equation) should be straightforward
- Convergence???

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# Thinking About The Reynolds Equation

- Can we make the element independent of fluid assumption (to the extent possible)?
  - Incompressible
  - Perfect gas
  - Real gas
  - Make it easy to add thermal effects, fancier turbulence models, etc. later
- Can we separate the geometry details?
  - Radial versus thrust versus slider
- OOFEM's design separates the material, cross-section, and element definitions for structural elements
- Can we do this for Reynolds?

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## Reynolds equation

- Start with Reynolds equation before assuming incompressible or perfect gas

$$\frac{\partial}{\partial x} \left( \rho H^3 \frac{1}{12\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho H^3 \frac{1}{12\mu} \frac{\partial P}{\partial y} \right) = \frac{U_x}{2} \frac{\partial}{\partial x} (\rho H)$$

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## Reynolds Equation

- Define a “material” property that is a function of pressure, local surface velocity, and film thickness!

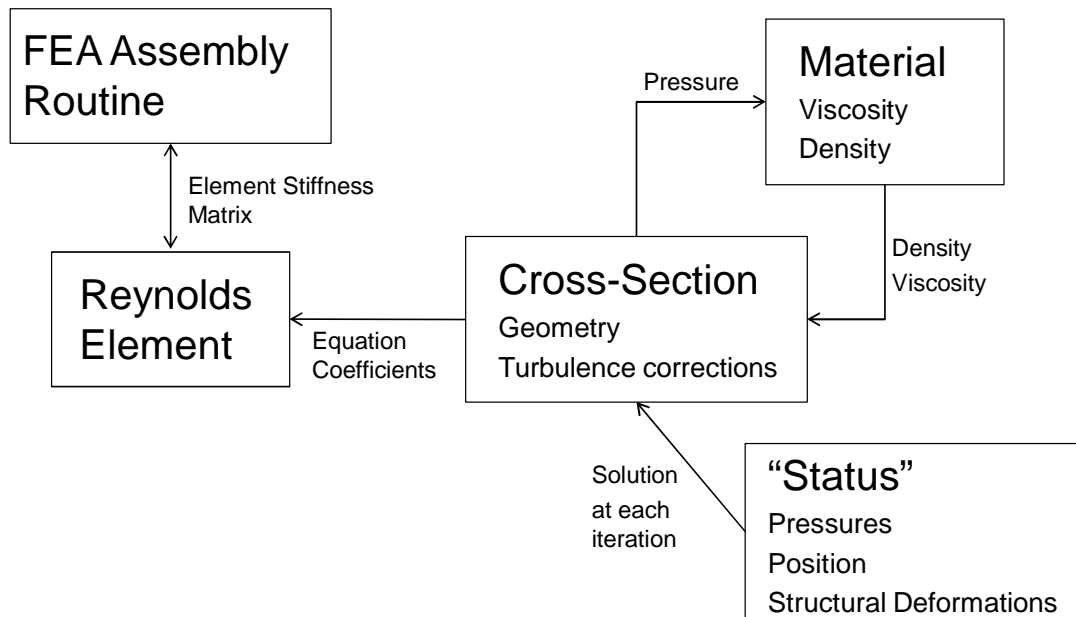
$$\frac{\partial}{\partial x} \left( \Gamma_x(P, H, \Omega) \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_y(P, H, \Omega) \frac{\partial P}{\partial y} \right) = \frac{U_x}{2} \frac{\partial}{\partial x} (\rho(P) H)$$

- Can put this “property” in one or more separate modules.
- Now the element does not care if it is compressible or not
  - Would need some additional work to handle cavitation
- Pretty straightforward to expand to include temperature

## Reynolds Element Formulation

- Three Solution Variables
  - Pressure (P)
  - Structural Displacements (U -> Element Film Thickness H)
  - Shaft Position (X -> Element Film Thickness H)
- The problem will be nonlinear
- Development of a Galerkin finite element proceeds as usual
  - Weak form
  - Integration by parts
  - More terms, since film thickness is also solution variable

# OOFEM Reynolds Implementation



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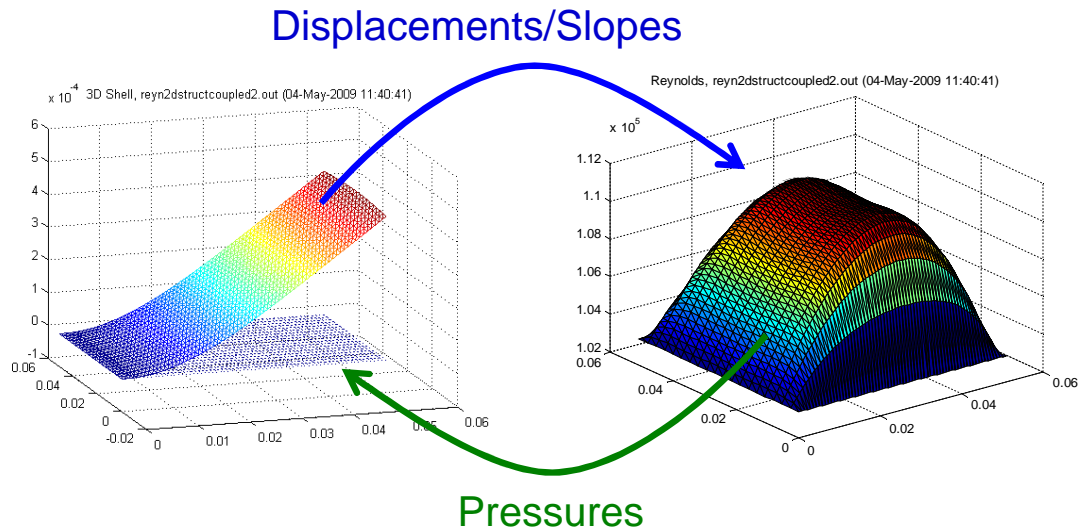
## Two Complications

- Need partial derivatives of the "material" property term with respect to pressure and film thickness in the element calculations
  - Handle with the chain rule for partial derivatives
  - Can maintain modular structure
- The coupling terms
  - Film thickness is a function of structural deformations
  - Structural deformations are functions of film pressures

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# Mesh Coupling



But, Meshes May Not Match!



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## Mesh Coupling – Mismatched Meshes

- Want to allow for mis-matched meshes between fluid and structural domains.
  - Structural domain XYZ coordinates, while fluid is polar RΘZ coordinates for radial and thrust bearings
  - Elements may differ shape, order, etc
- Need connection to be automatic, not require user intervention or significant pre-processing
- OOFEM has a “spatial locator” module that will determine which element in a particular mesh (domain) contains a specified point
  - Perfect!



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## Mesh Coupling - Interpolation

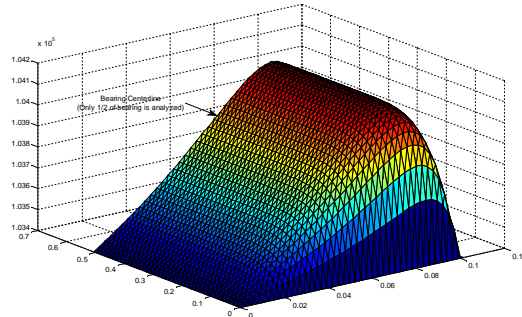
- Elements are defined with interpolating functions
- These give pressure (or displacement) at any location as weighted sum of nodal values
  - $P(xyz) = a_1*P_1 + a_2*P_2 + a_3*P_3 + \dots$
- Why not use these?
  - Conceptually pretty clean
  - Requires adding some bookkeeping capabilities to fluid and structural elements that include coupling to store the  $a_1, a_2, \dots$  and which nodal values to use
  - Works for slider, radial and thrust bearings pretty much the same way
  - Implantation remains modular
  - May be a bit simplistic though

## What About The Shaft Motion?

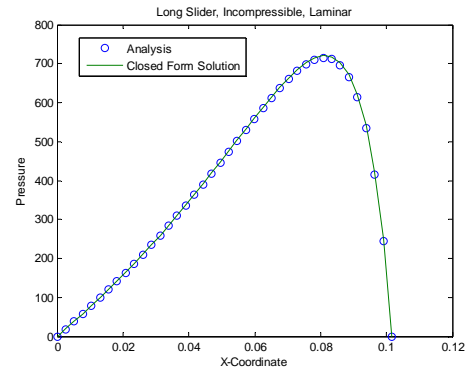
- Can add equations for a force balance on the shaft to the overall system
- Allows simultaneous solution for pressures, structural deformations and shaft position
- So far seems to converge pretty reliably
- Equation scaling is a key consideration for good numerical performance
  - Pressures  $\sim 1e5$  Pa
  - Displacements  $\sim 1e-4$  m
  - Rotations  $\sim$  very small
  - Need to rescale to get variables closer to all same order of magnitude

# Some Results – Reynolds Only

## Incompressible Tapered Slider



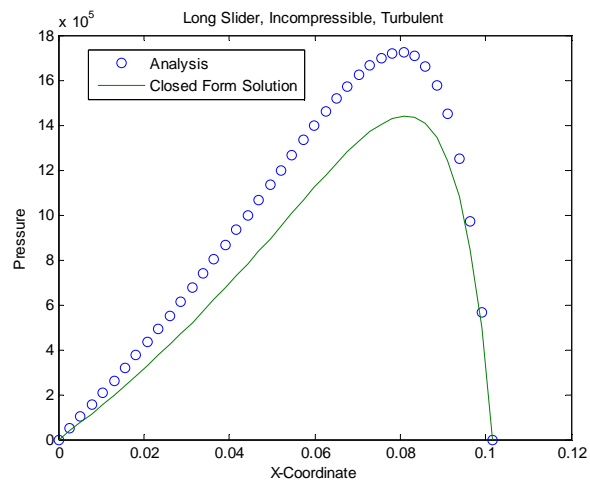
Full Pressure Field



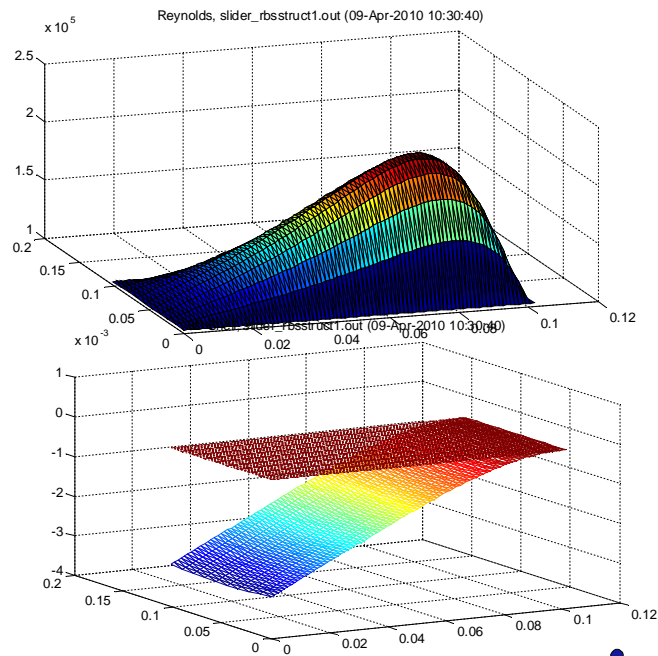
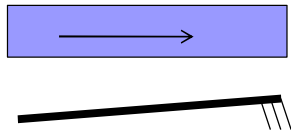
Centerline Comparison with Closed Form Solution

# Faster, More Turbulent

Checks, pressure with turbulence is higher than laminar closed form solution

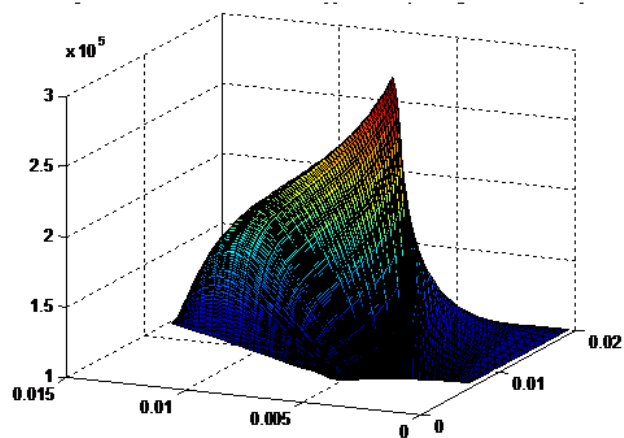
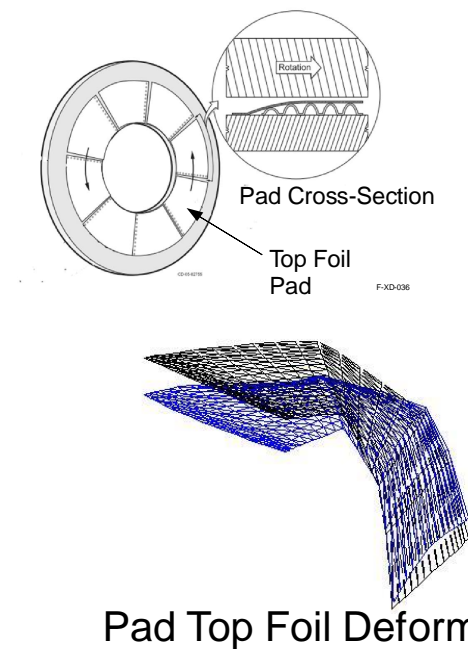


# Some Results – Coupled Slider/Beam



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# Some Results – Compliant Thrust Bearing

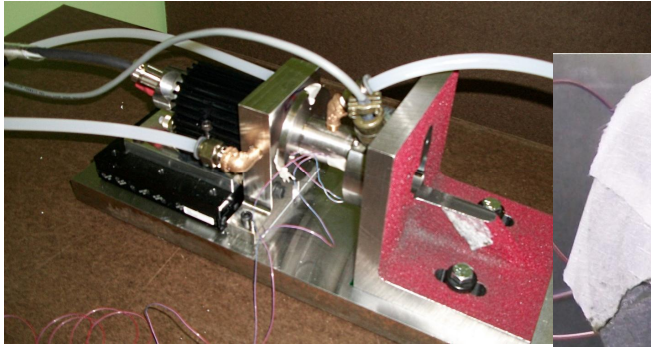


Pad Pressures  
(Compressible Fluid!)

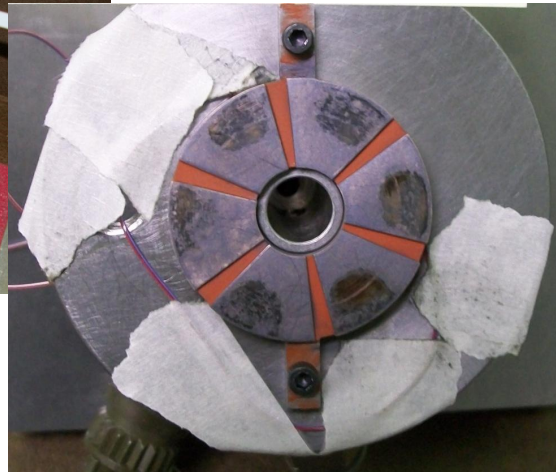


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# And, It Worked!



70 kRPM Thrust  
Bearing Test Rig



Bearing Post Test

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## Some Acknowledgements

- Initial development of code was supported as part of an Air Force Phase I SBIR Project.
- Development of Thrust Bearing analysis partially supported by a Navy Phase I STTR Project.
- Some of development was also “funded” as an internal Xdot research and development project.

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## Future Work

- Get coupled radial bearing working correctly
  - Problems in OOFEM shell element with drilling degrees of freedom for non-planar models
  - New element in current revision should fix this
- Stiffness and damping from perturbation solution
- Add thermal model to get temperatures
- “XFEM” (eXtended Finite Element Method) for cavitation
  - Allows arbitrarily located discontinuities that do not have to align to an element boundary
  - Sounds very promising for better cavitation model

## Summary/Observations

- Overall, taking this approach has worked pretty well.
- The learning curve was pretty steep.
  - It would have been really hard if I did not have a solid background in FEA theory and C++ programming.
- Using a framework avoided quite a bit of reinventing the wheel.
- Even with some added overhead from a very general purpose framework, it runs pretty quickly.
- As I add features, I keep finding things in the framework that already implement at least part of what I want to do.